Nonlinear Transient Simulation of Embedded Subnetworks Characterized by S-parameters Using Complex Frequency Hopping

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Abstract - *This paper describes an efficient technique for simulation of linear subnetworks characterized by s-parameters using general purpose circuit simulators such as SPICE. The proposed method is based on the recently developed model-reduction technique, complex frequency hopping (CFH). A new algorithm for computing the moments of s-parameter based subnetworks is presented and it overcomes the instability problems associated with the previously published techniques which depend on numerical differentiation. The proposed method is suitable for simulating large number of s-parameter based subnetworks in a general circuit environment consisting of lumped/distributed elements and nonlinear devices.*

I - INTRODUCTION

Recently, characterization and simulation of linear subnetworks based on scattering parameters has become a topic of intense research. Important applications of scattering parameters include high-frequency microwave devices and high-speed interconnects. At higher frequencies, it may not always be possible to have analytical models for interconnects due to the topological and inhomogeneity constraints. For example, in chip carriers, interconnections are usually nonuniform due to the high-circuit density, the complex geometry, and the geometrical constraints at the edges of the chip. Nonuniform transmission lines are also used as filters, couplers, impedance matching blocks, equalizers, resonators and pulse transformers. Generally the behavior of such structures are characterized using frequency-dependent s-parameters [1] - [8]. Such s-parameters can be obtained either directly from measurements or from rigorous full-wave electromagnetic simulation.

However, transient simulation of such frequency-dependent scattering parameters in the presence of nonlinear devices is a CPU expensive process. This can be attributed to the mixed frequency/time problem as the network contains both s-parameters which are characterized in the frequency-domain and nonlinear devices which are represented only in the time-domain.

There have been several attempts in the literature to address the above issue. These approaches can be broadly classified into two categories. In the first category transient simulation is performed based on the traditional convolution process [3], [4], [5] wherein the frequency-domain measured data is first converted to time-domain using inverse Fourier transform and then convoluted with the transient responses of both the nonlinear load and the input excitation. However, such an approach suffers in a general circuit environment containing large number of nonlinear devices due to computational inefficiency and convergence problems. Approaches in the second category are based on obtaining a reduced-order rational function approximation [6], [7], [8] for the measured data and performing the transient analysis using recursive convolution [7], [9]. However, there are three main difficulties associated with these approaches: (a) lack of a systematic approach in [6] to capture the entire frequency spectrum of interest from the given s-parameters, (b) instability problems associated with the computation of moments using numerical differentiation as suggested in [8] and (c) CPU expense and stability problems associated with the convolution technique.

In this paper, we describe an efficient and numerically stable method to address the above mentioned shortcomings. The main contributions/advantages in the new technique are summarized below:

(1) An algorithm based on the complex frequency hopping (CFH) [10] is presented for the model-reduction of the global circuit matrix consisting of s-parameter based subnetworks. This will reduce the size of the problem under consideration and results in two to three order CPU speed-up compared to previous techniques. *Use of the CFH algorithm helps to preserve the frequency spectrum of the s-parameters up to the highest frequency of interest*.

(2) A new moment-generation scheme based on time-domain integration is developed for s-parameter based subnetworks. *The proposed method overcomes the stability problems associated with the previous techniques which are based on numerical differentiation*.

(3) A time-domain macromodel from the reduced-order description is derived, which can be easily stencilled into general purpose simulators and can be simulated with any nonlinear devices to obtain time responses [11]. *The proposed scheme overcomes the problems associated with the convolution techniques*.

II - GENERALIZED FORMULATION OF LINEAR NETWORKS CONTAINING S-PARAMETER BASED DEVICES

In order to perform model-reduction on a linear network π comprising of lumped components and s-parameter based devices, MNA (modified nodal analysis) representation of the linear network π is required. This is accomplished by extending the MNA to include s-parameter based subnetworks. The stencil for s-parameter based subnetworks can be represented in terms of y-parameters as

$$\boldsymbol{I}_k(s) = \boldsymbol{Y}_k(s)\boldsymbol{V}_k(s) \qquad \textbf{(1)}$$

where $\boldsymbol{I}_k(s)$ and $\boldsymbol{V}_k(s)$ are the Laplace-domain terminal current/voltage vectors and $\boldsymbol{Y}_k(s)$ is a $N_k \times N_k$ y-parameter matrix for the measured subnetwork k. The y-parameters of the subnetwork k are related to the s-parameters as [1]

$$\boldsymbol{Y}_k\boldsymbol{F}_k(\boldsymbol{S}_k\boldsymbol{R}_k + \boldsymbol{R}_k^*) = \boldsymbol{F}_k(\boldsymbol{I}_k - \boldsymbol{S}_k) \qquad \textbf{(2)}$$

where

$$\boldsymbol{F}_k = diag\left[\frac{1}{2\sqrt{Re(Z_1)}} \quad \cdots \quad \frac{1}{2\sqrt{Re(Z_N)}}\right] \qquad \textbf{(3)}$$

$$\boldsymbol{R}_k = diag[Z_1 \quad \dots \quad Z_N] \qquad \textbf{(4)}$$

Here $Z_1 \dots Z_N$ are the reference impedances at the terminals of the subnetwork. $\boldsymbol{S}_k$, $\boldsymbol{I}_k$ are $N_k \times N_k$ s-parameter and identity matrices, respectively. '*' represents the complex hermitian. Next, using (1), the network equation in the frequency-domain can be written as

$$\begin{bmatrix} \boldsymbol{G}+s\boldsymbol{W} & \boldsymbol{E_1} & \boldsymbol{E_2} & \dots & \boldsymbol{E_{N_s}} \\ \boldsymbol{Y_1}\boldsymbol{E_1^t} & -\boldsymbol{U} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \dots & \boldsymbol{0} & \boldsymbol{0} & \dots & \boldsymbol{0} \\ \boldsymbol{Y_{N_s}}\boldsymbol{E_{N_s}^t} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{U} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}(\boldsymbol{s}) \\ \boldsymbol{I}_1(\boldsymbol{s}) \\ \dots \\ \boldsymbol{I}_{N_s}(\boldsymbol{s}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{0} \\ \dots \\ \boldsymbol{0} \end{bmatrix} \qquad \textbf{(5)}$$

where $\boldsymbol{W}, \boldsymbol{G} \in \Re^{N_\pi \times N_\pi}$ are constant matrices determined by lumped linear components, $\boldsymbol{b} \in \Re^{N_\pi}$ is a constant vector with entries determined by independent voltage/current sources, $\boldsymbol{V}(\boldsymbol{s}) \in \Re^{N_\pi}$ is the vector of node voltage waveforms appended by inductor/independent voltage source currents of subnetwork π. $\boldsymbol{E}_k$ is a selector matrix with entries $e_{i,j} \in \{0, 1\}$ that maps $\boldsymbol{I}_k(\boldsymbol{s}) \in \Re^{N_k}$, the vector of currents entering the subnetwork k, into the node space of network π. N_s is the total number of measured subnetworks.

III. MODEL-REDUCTION TECHNIQUES FOR UNIFIED TRANSIENT SIMULATION

The difficulty in simulating (5) in the presence of nonlinear devices is due to the fact that they implicitly contain a mixture of frequency/time descriptions. This can be efficiently addressed with the help of recently developed model-reduction techniques as follows.

Using complex frequency hopping [10], the terminal behavior of the linear network π is approximated by a q- pole lower order model. Let n_π be the number of external terminals and $\boldsymbol{Y}_\pi(s)$ be the terminal y-parameter matrix associated with the linear network π. The reduced-order model can be obtained by driving external terminals of the linear network one at a time by an impulse source and obtaining corresponding columns of $\boldsymbol{Y}_\pi(s)$ through model-reduction on the MNA matrix given in (5). The reduced-order model for $\boldsymbol{Y}_\pi(s)$ is represented as

$$\begin{bmatrix} Y_{11} & \cdots & Y_{1n_\pi} \\ \cdots & \cdots & \cdots \\ Y_{n_\pi 1} & \cdots & Y_{n_\pi n_\pi} \end{bmatrix} \begin{bmatrix} V_1 \\ \cdots \\ V_{n_\pi} \end{bmatrix} = \begin{bmatrix} I_1 \\ \cdots \\ I_{n_\pi} \end{bmatrix}; \quad Y_{jk}(s) = d^{j,k} + \sum_{i=1}^{q^{j,k}} \frac{r_i^{j,k}}{s - p_i^{j,k}} \qquad \textbf{(6)}$$

where $1 \le (j,k) \le n_\pi$. $p_i^{j,k}$ is the i^{th} dominant pole at a port k due to an input excitation at port j and the corresponding residue is $r_i^{j,k}$. $d^{j,k}$ is the direct coupling constant. $q^{j,k}$ is the number of dominant poles used for approximating H_{jk}. *Use of the CFH algorithm helps to preserve the frequency spectrum of the s-parameters up to the highest frequency of interest*. However, model-reduction on the MNA matrix (5) requires moments of s-parameter based components. In order to address this, a new moment-generation algorithm has been developed.

IV. NEW MOMENT-GENERATION ALGORITHM AND TRANSIENT SIMULATION IN THE PRESENCE OF NONLINEAR DEVICES

In order to apply the CFH algorithm, moments of the individual entries of the MNA matrix in (5) are needed. However, evaluation of moments $\boldsymbol{Y}_k^{(\boldsymbol{r})}(s)$ of s-parameter based linear subnetworks in the existing literature is not adequately efficient for CFH based simulation. In the following a brief discussion of the new moment-generation method for subnetworks which are characterized by s-parameters is presented. Derivatives of (5) can be recursively computed (details of which is not given here due to the lack of space), and it leads to the task of computing the moments of the table of data representing the s-parameters. However, s-parameters in this form do not have a closed-form solution. In order to address this, s-parameters are first converted to time-domain using inverse Fourier transform. Next, the frequency-domain derivatives of s-parameters are obtained by performing integration in the time-domain. For the purpose of illustration, consider any individual s-parameter, $\boldsymbol{s}_{ij}(t)$ in the time-domain. Using integration in time-domain, derivatives of $\boldsymbol{S}_{ij}$ in frequency-domain can be computed as

$$\frac{d^r}{ds^r}\boldsymbol{S}_{ij}(s) = \int_0^\infty (-t)^r \boldsymbol{s}_{ij}(t) e^{-st} dt \qquad \textbf{(7)}$$

Derivatives represented by (7) do not suffer from the ill-conditioning problems which are usually associated with the numerical differentiation based moment-computation methods.

Extraction of a lower-order description through model-reduction techniques facilitates efficient simulation of the given linear network π in the presence of nonlinear components. This can be achieved by deriving a state-space representation in time-domain from the reduced-order model of the linear subnetwork. These differential equations can be easily stenciled into general purpose simulators like any other circuit components and simulated with nonlinear devices [11].

V - COMPUTATIONAL RESULTS

An example is given below to demonstrate the accuracy of the proposed technique. The network contained six transmission lines which are characterized by s-parameters and two inverters. The proposed CFH algorithm is used to obtain a reduced-order model for the entire linear network. Moments of the s-parameter based subnetworks are obtained using the time-domain integration scheme proposed in this paper. Next, a macromodel in terms of state-space variables is derived from the reduced-order model and a transient simulation is performed by combining the macromodel with the nonlinear elements using SPICE. Accuracy of the results from the proposed algorithm is compared with the transient simulation of the original network using SPICE, in which each subnetwork is replaced by its quasi-TEM analytical models. Responses from both the methods are given in Fig. 1 and they match accurately.

VI - CONCLUSIONS

In this paper an efficient technique based on model-reduction is presented for simulation of embedded linear subnetworks characterized by scattering parameters in the presence of nonlinear components. A new algorithm for moment-generation of s-parameter based subnetworks is described. The proposed technique provides an efficient means for simulating scattering parameters using general purpose circuit simulators.

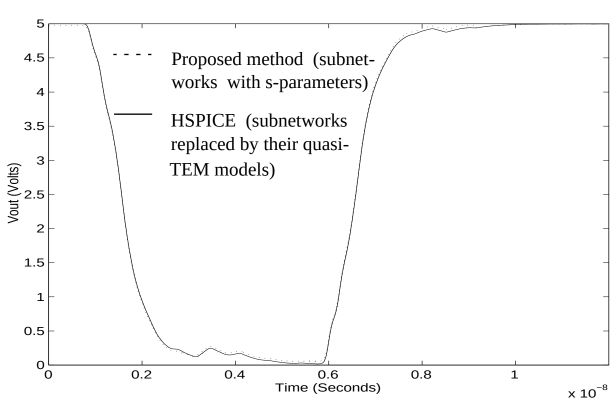

Fig. 1. Time response

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